WORKSHOP ON SYMPLECTIC TOPOLOGY: ABSTRACTS

Filip Broćić: Arnol'd's chord conjecture for conormal Legendrian lifts

Abstract. The chord conjecture, due initially to Arnol'd in the case of the standard contact three-sphere, asserts the existence of a Reeb chord with boundary on every closed Legendrian submanifold of a closed contact manifold for every contact form. This conjecture was established in various settings by Cieliebak, Mohnke, Hutchings and Taubes, and others. In this talk, I will sketch a proof of the chord conjecture for conormal bundles of closed submanifolds of any closed manifold seen as Legendrians in the co-sphere bundle. This generalizes a result of Grove in Riemannian geometry regarding the existence of geodesics normal to the submanifold. The method of proof involves wrapped Floer cohomology with local coefficients. This talk is based on a joint work in progress with Dylan Cant and Egor Shelukhin.

Lev Buhovsky: Groups of area-preserving homeomorphisms, spectral estimators, and Sikorav's trick

Abstract. The celebrated Fathi question asked about the simplicity of the group of Hamiltonian homeomorphisms of a symplectic surface. The recent solution of the question introduced the group of finite energy Hamiltonian homeomorphisms which was shown to be a non-trivial normal subgroup, thus giving the negative answer to the question. That group of finite energy Hamiltonian homeomorphisms contains in itself the group of Oh-Müller Hamiltonian homeomorphisms. In my talk I will try to explain how one can compare between these groups and show that their quotient is large from the perspective of Hofer's geometry.

Yaniv Ganor: Persistence in Wrapped Floer Homology and Poisson Bracket Invariants

Abstract. The Poisson bracket invariants, introduced by Buhovsky, Entov and Polterovich, are invariants of quadruples of closed sets in symplectic manifolds. Their nonvanishing has implications to the existence of Hamlitonian trajectories between pairs of the sets in the quadruple.

In this talk, we will describe a work in progress, where we obtain lower bounds of the Poisson bracket invariants of certain configuration arising in the completion of Liouville manifolds, in terms of the barcode of wrapped Floer homology (we will give all necessary definitions in the talk). This work is inspired by Entov and Polterovich, who proved similar results for Lagrangian cobordisms between two contact manifolds, using persistence in Legendrian contact homology.

Our main examples are cotangent bundles of Riemanninan manifolds, where the quadruple comprises two cosphere bundles of different radii, and two fibers above different points. In this example the nonvanishing of the Poisson bracket invariant

implies the existence of Hamiltonian trajectories going between the fibers, with an explicit bound on their time-length, for certain Hamiltonian perturbations of the geodesic flow.

Dušan Joksimović: On generalizations of Gromov-Eliashberg's theorem

Abstract. Symplectic structure on a smooth manifold is given by a differential 2-form that is closed and non-degenerate. A famous theorem of Y. Eliashberg and M. Gromov states that the group of diffeomorphisms which preserve a given symplectic structure (i.e. the symplectomorphisms group) forms a closed subset inside the group of all diffeomorphisms equipped with the compact-open topology.

In this talk we will try to emphasize that non-degeneracy is not necessary for obtaining C^0 rigidity in the above sense. In that direction, we will prove Eliashberg-Gromov's C^0 rigidity for Poisson manifolds. More precisely, we will prove that the group of Poisson diffeomorphisms forms a closed subset inside the group of all diffeomorphisms equipped with the compact-open topology. If time permits, we will mention another generalization of Eliashberg-Gromov's theorem for presymplectic manifolds which is based on a work in progress with Kai Cieliebak and Fabian Ziltener.

Rémi Leclercq: About the topology of the C^0 and spectral completions of Ham

Abstract. I will explain recent progress in understanding the fundamental group of the C^0-completion of Hamiltonian diffeomorphism groups. The spectral norm of Hamiltonian loops, computed via the Seidel representation, detects non-trivial classes in the fundamental group of the spectral completion of Ham. When the spectral norm is proved to be C^0 continuous, those classes yield non-trivial classes in the C^0-completion. As an illustration, we exhibit explicit non-trivial elements in the case of $\mathbb{C}P^n$ (C^0-completion) and all rational 1-point blow-ups of $\mathbb{C}P^2$ (spectral completion). This is joint work with Vincent Humilire and Alexandre Jannaud.

Stefan Matijević: $\mathbb{S}^1$-equivariant Floer chain complex and Clarkes duality

Abstract. Periodic orbits of a convex Hamiltonian on $\mathbb{R}^{2n}$ can be studied by the direct action functional and by Clarkes dual action functional. Comparing these two approaches, we show that the $\mathbb{S}^1$-equivariant Floer complex associated with an autonomous convex Hamiltonian function is isomorphic to the $\mathbb{S}^1$-equivariant Morse complex of Clarkes dual action functional associated with the Fenchel-dual Hamiltonian. Furthermore, this isomorphism preserves action filtrations.

Branislav Prvulović: Lyusternik–Shnirel'man category and the cup-length of Grassmann manifolds

Abstract. Lyusternik–Shnirel'man category of the space X (denoted by $\mathrm{cat}(X)$) is the minimal positive integer d such that X can be covered with d open subsets each of which is contractible in X. This homotopy invariant of a space is very hard to compute in general. A well-known lower bound for $\mathrm{cat}(X)$ is the cup-length

of the cohomology algebra of the space X (with arbitrary coefficients R). It is defined as the supremum of the set of all integers m such that there exist positive dimensional cohomology classes $a_1, a_2, \ldots, a_m \in H^*(X; R)$ with the property that the cup product $a_1 a_2 \cdots a_m$ is nontrivial.

We will present a proof of a conjecture made by Tomohiro Fukaya in 2008, which concerns the cup-length of the Grassmann manifolds $\widetilde{G}_{n,3}$ of oriented 3-dimensional subspaces in $\mathbb{R}^n$. The main techniques of the proof include the theory of Gröbner bases, Poincaré duality and some other classical methods of algebraic topology.

Dietmar Salamon: Tame complex structures, moment maps, and the Calabi program

Abstract. The purpose of this talk is to explain recent work of Vestislav Apostolov, Jeffrey Streets, and Yury Ustinovskiy in which they extend the Calabi program for the study of constant scalar curvature and extremal Kaehler metrics to a setting where the complex structure is no longer compatible with the symplectic form but only tamed by it. Their work includes the construction of analogues of the space of Kaehler potentials and the Mabuchi functional, in the tame setting.

Matija Srećković: Floer Theory of Lefschetz Fibrations on Cotangent Bundles Extending Morse Functions

Abstract. In this talk, I will discuss a conjectural link between the flow category of a Morse function f on a closed manifold M and the directed Donaldson-Fukaya category of a Lefschetz fibration on T^*M whose restriction to the zero section is equal to f. This construction is due in some special cases to Johns, and the general case has been carried out in recent work of Giroux. I will briefly sketch this construction, state the conjecture explicitly, and finally describe my ongoing work on the conjecture in the three-dimensional case, where f is a Morse function on an oriented manifold inducing a Heegaard diagram.

Vukašin Stojisavljević: Persistent transcendental Bézout problem

Abstract. Transcendental Bézout problem concerns the count of zeros of holomorphic self-maps of complex vector spaces. A classical question asks if it is possible to bound from above the number of zeros inside a ball of a certain radius in terms of the maximum modulus of the function on a slightly larger ball. In 1972 Cornalba and Shiffman provided examples showing this to be impossible in dimensions greater than one. In contrast, we will prove that predicted upper bounds hold for the coarse count of zeros. This count has roots in topological data analysis, namely in the theory of persistence modules and barcodes. The talk is based on a joint work with L. Buhovsky, I. Polterovich, L. Polterovich and E. Shelukhin.

Maksim Stokić: Adjoint action and Hofer's geometry on the group of Hamiltonian diffeomorphisms

Abstract. The space of Hamiltonian diffeomorphisms has a structure of an infinite dimensional Frechet Lie group. Its Lie algebra is isomorphic to the space of zero-mean normalized functions, and the adjoint action is given by pull-backs. We show that this action is flexible: for a non-zero normalized function u, any other normalized function f can be written as a finite sum of elements in the orbit of u under the adjoint action. Moreover, the number of elements in the sum is dominated from above by the uniform norm of f. We will discuss implications in Hofer's geometry and recent progress in the case of open symplectic manifolds. This is joint work with Lev Buhovsky.

Umut Varolgunes: Involutive covers and closed string mirror symmetry

Abstract. I will recall the notion of an involutive cover of a symplectic manifold and the local-to-global principle which shows the existence of a spectral sequence starting from the relative symplectic cohomology of the cover and ending at quantum cohomology. I will then discuss the compatibility of this spectral sequence with natural algebraic structures that are present. This leads to a research program that aims to explain genus 0 closed string CY mirror symmetry in a purely symplectic manner, which I will sketch at the end of my talk.

Jun Zhang: New approaches to discovering symplectic non-convexity

Abstract. In this talk, we will provide new examples of star-shaped (toric) domains in $\mathbb{C}^2$ that are dynamically convex but not symplectically convex. Our examples are based on two approaches: one is from Chaidez-Edtmair's criterion via Ruelle invariant and systolic ratio; the other is from the ECH capacities and an analog non-linear version of Banach-Mazur distance in symplectic geometry. In particular, from the second approach, we derive the first family of examples that can be numerically verified (instead of taking a certain limit from the first approach). We will also illustrate that the information given by these two approaches is in general independent of each other. This talk is based on joint work with Dardennes, Gutt, and Ramos.

Frol Zapolsky: Restrictions on symplectic quasi-states

Abstract. Quasi-states are certain, usually nonlinear, functionals on the space of continuous functions of a topological space. When the space in question is a symplectic manifold, a more restrictive notion of symplectic quasi-states was introduced by Entov-Polterovich, who also invented the only general method of constructing them known to-date, which uses tools of hard symplectic topology. In dimension two the two notions coincide, and thus (symplectic) quasi-states can be constructed using soft techniques. In the talk I'll explore the question whether these techniques can be used to construct nonlinear symplectic quasi-states in higher dimensions, as well as some other restrictions on these. Based on joint work with Adi Dickstein.

Fabian Ziltener: A relative Hofer estimate and the asymptotic Hofer-Lipschitz constant

Abstract. This talk is about joint work with Michael Khanevsky.

Let (M, ω) be a symplectic manifold and U an open subset of M. We study the natural inclusion of the compactly supported Hamiltonian group of U in the compactly supported Hamiltonian group of M. The main result is an upper bound for this map in terms of the Hofer norms for U and M.

Applications are upper bounds on the asymptotic Hofer-Lipschitz constant and the relative Hofer diameter of U. The first bound is often sharp and the second one is often sharp up to a factor of 2.

Filip Živanović: Symplectic $\mathbb{C}^*$-manifolds

Abstract. I will define a broad family of open symplectic manifolds admitting pseudoholomorphic $\mathbb{C}^*$-actions, which contains many interesting spaces, such are equivariant resolutions of affine singularities, twisted cotangent bundles, semiprojective toric varieties and Higgs moduli. I will explain how one can construct symplectic cohomology and spectral sequences converging to it, although the symplectic structures on these spaces are typically highly non-exact at infinity. As a consequence, we get a filtration on their quantum cohomology rings by ideals, which should be thought of as a Floer-theoretic analogue of Atiyah–Bott filtration. This filtration is in practice computable using the aforementioned spectral sequences, which we do for twisted cotangent bundles, ADE resolutions, certain Slodowy varieties and certain parabolic Higgs moduli spaces. For the last ones, we compare it with the "$P = W$" filtration. This is joint work with Alexander Ritter.