WORKSHOP ON SYMPLECTIC TOPOLOGY: ABSTRACTS

Filip Broćić: A counterexample to Lagrangian Poincaré recurrence

Abstract. Poincaré recurrence is a classical result in measure-preserving dynamics, which guarantees that almost every point of a set of positive measure comes back to the set for infinitely many iterations. Lagrangian Poincaré recurrence (LPR) is a conjecture in Hamiltonian dynamics which states that for every closed symplectic manifold M, closed Lagrangian submanifold L, and Hamiltonian diffeomorphism F, the set of integers k such that $F^k(L)$ intersects L must be infinite (despite L being of measure zero). In the talk, I will explain the construction of a counterexample to the LPR in every symplectic manifold of dimension at least 6. The talk is based on a joint work with Egor Shelukhin.

Octav Cornea: Approximability for Lagrangians

Abstract. Turing introduced in 1938 a notion of approximability for metric groups that has been later studied in geometric group theory. In this talk I will discuss a categorified version of this notion and its application to rigidity properties and complexity measurements of Lagrangian submanifolds. The talk is based on work in progress with Giovanni Ambrosioni and Paul Biran.

Dušan Drobnjak: Exotic symplectomorphisms in $\mathcal{O}(-1)$

Abstract. I will present a criterion for detecting exotic symplectomorphisms in strong fillings of closed contact manifolds. This criterion is formulated in terms of the existence of a free contact circle action on the boundary, satisfying certain topological conditions. As an application, exotic symplectomorphisms are explicitly constructed in the total space of the tautological line bundle over $\mathbb{C}P^n$, commonly denoted by $\mathcal{O}(-1)$. The corresponding symplectic mapping classes are of finite order and, in certain dimensions, exotic symplectomorphisms are not smoothly exotic.

Milica Đukić: A deformation of the Chekanov-Eliashberg algebra using pseudoholomorphic annuli

Abstract. Symplectic field theory (SFT) is a powerful framework for studying contact manifolds and their Legendrian submanifolds. In this talk, we introduce an SFT-type invariant for Legendrian knots in $\mathbb{R}^3$, which is a deformation of the Chekanov-Eliashberg dga

that incorporates contributions from pseudoholomorphic annuli, and describe a combinatorial way to compute it from the Lagrangian projection. We additionally show that this invariant computes the coproduct on the linearized contact homology of the contact manifold obtained by attaching a Lagrangian handle along a Legendrian knot.

Yakov Eliashberg: Distinguishing open contact manifolds and related problems

Abstract. There are not many tools for distinguishing open contact manifolds. For instance, for $n > 1$ it is not known whether any diffeomorphic to $\mathbb{R}^{2n+1}$ open submanifold of the standard contact $\mathbb{R}^{2n+1}$ is contactomorphic to it (for $n = 1$ the answer is positive). In the talk I will exhibit some continuous families of non-contactomorphic open manifolds and discuss tightly related problems of contact squeezing and encompassing. This is a joint work with Kiran Ajij, Mahan Mj, Dishant Pancholi and Leonid Polterovich.

Yaniv Ganor: Torsion in Relative Symplectic Cohomology, and a New Rigidity Phenomenon

Abstract. Relative symplectic cohomology, introduced by Varolgunes, assigns to each compact subset of a symplectic manifold a module in a functorial way. While its torsion-free part has been computed in several cases, leading to new rigidity results, the torsion component, carrying quantitative information, remains largely unexplored, reflecting the complexity of the construction.

In this talk, based on joint works in progress with Adi Dickstein and Frol Zapolsky, we investigate several aspects of torsion in relative symplectic cohomology. We provide a complete computation for balls in $\mathbb{CP}^n$, and we study a relative symplectic capacity defined via the torsion exponents in relative symplectic cohomology, exploring its properties and relationship with the Gromov width.

As an application we demonstrate a new type of symplectic rigidity phenomenon, detected by the torsion in relative symplectic cohomology.

Dušan Joksimović: Lipschitz homeomorphisms in symplectic and contact geometry

Abstract. Eliashberg-Gromov's famous C^0 rigidity led to definitions of symplectic and contact homeomorphisms and opened a new area of research (known as C^0 symplectic geometry), which led to many interesting results in symplectic/contact geometry and dynamics. In this talk, we will focus on the class of Lipschitz homeomorphisms in symplectic and contact geometry, providing an overview of known results and open questions related to these classes of maps.

Danica Kosanović: Diffeomorphisms from dancing circles

Abstract. It is still unknown whether the 4-sphere has trivial smooth mapping class group (diffeomorphisms modulo isotopy). Recently, Gay has shown that any potential class can be realised using a 1-parameter family of embedded 2-spheres. In fact, we showed that all candidate classes constructed so far come from 1-parameter families of embedded circles, and often reduce to a single class. Analogues of such families are nontrivial in some non-simply connected 4-manifolds, as shown by Budney-Gabai and Watanabe . In this talk I will first explain how one can study families of circles, and show that they give rise to nontrivial diffeomorphisms of $S^1 \times S^2 \times [0, 1]$, as done in the joint work with E. Fernández, D. T. Gay, and D. Hartman.

Vladimir Marković: Realization problems for mapping class groups

Abstract. Let G denote a subgroup of the mapping class group of a surface S. The classical problem is whether G admits a lift to the groups of homeomorphisms $\mathrm{Homeo}(S)$, or diffeomorphisms $\mathrm{Diff}(S)$, is known as the Realization Problem. When G is a finite group this probelm is known as the Nielsen Realization Problem. Obstructions to lifting G to $\mathrm{Diff}(S)$ are known as Mumford-Miller-Morita characteristic classes. Finally, topological dynamics is the key tool in studying the lifting problem to $\mathrm{Homeo}(S)$. I will discuss all this, including the most recent results concerning the realization of finite index subgroups of the mapping class groups into the group of area preserving homeomorphisms.

Stefan Matijević: Systolic S^1-index and characterization of non-smooth Zoll convex bodies

Abstract. We define the systolic S^1-index of a convex body as the FadellâĂŞRabinowitz index of the space of centralized generalized systoles associated with its boundary. We show that this index is a symplectic invariant. Using the systolic S^1-index, we propose a definition of generalized Zoll convex bodies and prove that this definition is equivalent to the usual one in the smooth setting. Moreover, we show how generalized Zoll convex bodies can be characterized in terms of their Gutt-Hutchings capacities and we prove that the space of generalized Zoll convex bodies is closed in the space of all convex bodies. As a corollary, we establish that if the interior of a convex body is symplectomorphic to the interior of a ball, then such a convex body must be generalized Zoll, and in particular Zoll if its boundary is smooth. Finally, we discuss some examples.

Yong-Geun Oh: Contact Hamiltonian dynamics and geometric analysis

Abstract. In this talk, we will first explain the generic characteristics of contact Hamiltonian dynamics, and explain its quantitative aspects in relation to thermondynamics and contact topology. We then discuss geometric analysis of a new analytical machinery of contact instantons and its Hamiltonian perturbations, and several applications thereof to the study of the quantitative contact topology and others.

Alvaro del Pino Gómez: The h-principle fails for prelegendrians

Abstract. Given a pair of numbers (k, m), one can study maximally non-integrable distributions of rank k in m-dimensional manifolds. The most studied case is $(2n, 2n+1)$, Contact Topology. The other classic case is $(2n-1, 2n)$, the study of even-contact structures, whose homotopy classification was settled by McDuff. Other cases (e.g. (2,4), (2,5), (4,6),...) have been studied in the last few years from the perspective of the h-principle. Often one studies not just the structures themselves, but also the submanifolds that interact nicely with them (typically being transverse or tangent).

In this talk I will discuss a rigidity phenomenon in the study of (4,6) distributions (as far as I know, the first such result outside of contact topology). Namely, I will exhibit a family of submanifolds in the "standard elliptic (4,6) distribution" in $\mathbb{R}^6$, all of which are distinct (up to homotopy) despite being formally equivalent. That is, the h-principle fails for these submanifolds (which we call prelegendrians). This is joint work with Wei Zhou and Eduardo Férnandez.

Leonid Polterovich: Contact topology meets thermodynamics

Abstract. I discuss the appearance of certain notions and results from contact topology in both equilibrium and non-equilibrium thermodynamics. These include non-smooth Legendrian submanifolds, Reeb chords, and the partial order on the space of Legendrians. Based on joint work with Michael Entov and Lenya Ryzhik.

Vukašin Stojisavljević: Harmonic transcendental Bézout problem

Abstract. Transcendental Bézout problem concerns counting zeros of holomorphic maps between complex vector spaces of dimension greater than one. In 1972, disproving a classical prediction, Cornalba and Shiffman gave examples of holomorphic maps with slow growth of the modulus and fast growth of the number of zeros. I will discuss analogous examples in the realm of harmonic maps between real vector spaces.

Maksim Stokić: C^0-Contact Geometry of Surfaces in 3-Manifolds

Abstract. We present recent joint work with Baptiste Serraille. We prove a rigidity result for characteristic foliations on surfaces, showing that they are preserved under contact homeomorphisms. In contrast, we demonstrate that convex surfaces exhibit C^0-flexibility: we construct a contact homeomorphism sending a convex torus to a non-convex one.

Frol Zapolsky: Hofer-continuous quasi-morphisms for Liouville manifolds

Abstract. A quasi-morphism on a group is a real-valued function whose failure to be a homomorphism is bounded. Quasi-morphisms on infinite-dimensional groups of transformations have been the subject of substantial interest in the past three decades. In the case of the group of Hamiltonian diffeomorphisms, various instances of these functions exist. On one hand, there are geometric constructions, while on the other hand, we have quasi-morphisms coming from Floer-homological spectral invariants. Some of these apply to Liouville domains such as Euclidean balls and codisk bundles, and in these cases the defect of the quasi-morphisms, that is the failure to be a homomorphism, is always proportional to the size of the domain-either its volume or a symplectic capacity. This led L. Polterovich to conjecture that, for instance, the Hamiltonian group of a cotangent bundle should carry no quasi-morphisms at all, save the Calabi homomorphism. In the talk, I'll present my recent result regarding this conjecture under the additional assumption of Hofer continuity of the quasi-morphism in question.

Fabian Ziltener: C^0-rigidity of coisotropic embeddings of a presymplectic manifold

Abstract. This talk is about joint work in progress with Kai Cieliebak and Dušan Joksimović. The main result is that the set of coisotropic embeddings of a closed presymplectic manifold into a given presymplectic manifold is closed in the space of all embeddings with respect to the C^0-topology. This result generalizes several previously known forms of C^0-rigidity in symplectic geometry.

Filip Živanović: Filtrations, Hilbert Schemes and a search for Mirror Symmetry

Abstract. In this talk, I will survey joint works with A. Ritter, S. Szabó, and work in progress with A. Minets. The topic is understanding the interplay of symplectic topology and algebraic geometry of spaces given as Hilbert Schemes of certain holomorphic-symplectic surfaces, through a comparison of three different cohomological filtrations on them. These spaces are known to be Higgs bundles moduli spaces, so there is a natural quest to understand mirror symmetry in the sense of Kapustin-Witten for them, i.e. the duality between BAA

and BBB branes. The former branes naturally arise from our work, but the latter yet remain to be understood.